Vibration Analysis of Bent Pipes Using FEM

Authors

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ABSTRACT

Voluminous literature is available on the dynamic analysis of straight circular pipes. Similar analyses of bent pipes have not received wide attention in the research community. However, in most engineering applications pipeline networks generally contain one or more bends. In this article, we investigate the effect of bending radius on the natural frequencies of thin-walled pipes of circular cross-section using Finite Element Method (FEM). The FEM computation is accomplished using ANSYS. The boundary conditions at the ends are kept fixed. The modal analysis results are analyzed so as to determine the trend in natural frequency for a fixed mode of vibration for different bent radii. The objective of the analysis is to be able to infer from the results the effect of bend in terms of additional mass or stiffness. This facilitates better understanding of the curved shell dynamics. These results can also serve as benchmarks for further experimental or analytical investigations.

INTRODUCTION

Piping networks are commonly used in various industrial applications for conveying fluids and gases. Common examples of such applications occur in chemical industry, nuclear installations, aircraft fuselage, ventilation ducts, etc. Such piping networks mostly consist of straight pipes of annular cross-section with small thickness. However, to allow the fluid flow in different directions it also has bent portions. These bent portions join in to the straight pipes.

Vibration of such piping system is of great interest to the engineering community. Failures have been known to occur in such systems in response to dynamic loads. Further, condition monitoring systems for fault detection in these structures have evolved based on these vibration studies. Vibration prediction of straight pipes usually employs the well established thin shell theory.
There exists voluminous literature on straight pipes modelled as thin circular cylindrical shells. A comprehensive review of this material has been carried out by Leissa [1]. Similar analyses of bent pipes have not received wide attention in the research community.

Wang et al [2] has formulated the equations of motion or bent pipe in toroidal coordinate system using Sanders shell theory. Further, numerical results were presented and compared with earlier published work on Donell-Mushtari-Vlasov shell theory [Ref], as well as with Finite Element Method (FEM) results.

Salley [3] has demonstrated the efficacy of using FE analysis for similar pipe geometries by comparing with experimental results.

In the present work, we wish to study the effect of bend in thin-walled circular cylindrical shells. For this we use Finite Element Analysis through the commercial package ANSYS. The variation of natural frequencies with increasing bend angles is presented. The results shown are for two modes of vibration. The analysis shows the effect of bend angle is in terms of added stiffness for certain modes of vibration whereas for some other modes the effect of bend can be interpreted as an added mass on the straight cylindrical shell. Further, the value of added stiffness or mass is dependent on the mode of vibration.

**METHOD**

The commercial FEM program ANSYS is used in this study to determine the natural frequencies of straight and bent pipes as described above (see figure 1a and 1b). For the analysis we use quadrilateral 4 noded shell elements (SHELL63 in ANSYS). It has both bending and membrane capabilities. The model parameters used in the analysis is shown in Table 1. The validation process of the FEM model is described in the Appendix.

![Figure 1](image_url)

**Figure 1:** Geometry of the models used for analyses shown in orthographic projection (a) straight pipe (b) bent pipe.

Modal analysis is performed for different bend angles of the pipe keeping both ends as fixed (all dofs at the ends are constrained to zero). All other parameters indicated in Table 1 is kept constant.

**Table 1: Parameters used in modelling.**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of pipe</td>
<td>3m</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>2.1×10^11 N/m²</td>
</tr>
<tr>
<td>Thickness of the cross section</td>
<td>5mm</td>
</tr>
<tr>
<td>Density</td>
<td>7850 kg/m³</td>
</tr>
<tr>
<td>Radius of the cross section</td>
<td>10cm</td>
</tr>
<tr>
<td>Poisson’s ratio</td>
<td>0.3</td>
</tr>
</tbody>
</table>
It is well known that for straight circular cylindrical thin shells the modes of vibration of a cross-section is in the form of \( \cos(n \theta) \) (where \( n \) is any integer and \( \theta \) is the circumferential coordinate) \cite{2}. Schematic presentations of these modes are shown in Figure 2. Further, in the above reference it is shown that the ordering of the natural frequencies corresponding to these modes is not strictly monotonically increasing with \( n \). Specifically the natural frequency for the \( n=0 \) mode may be higher than the natural frequencies for the \( n =1 \) and the \( n = 2 \) mode. This is attributed to the interplay of membrane and bending effects in the shell geometry.

![Figure 2: Cross-sectional view of the mode shapes of straight circular cylindrical thin shells. The mode shapes are in the form of \( \cos(n \theta) \), \( n \) being an integer (a) \( n = 0 \), (b) \( n = 1 \), (c) \( n = 2 \).](image)

As said earlier, in the present work the objective is to investigate the primary effect introduced due to the bend in straight circular pipes. For this, the analysis is limited to small bend angles in the range of 1 degree to 10 degree in steps of one degree. Further, it is verified from the simulation results that the modal deflection pattern of the cross-section of bent pipes remain approximately same as in straight pipes \( \text{viz}. \) of the form \( \cos \left( n \frac{\pi}{L} \right) \) (as shown in figure 2). This is attributed to the small bend angle. For higher bend angles even the mode shape will change. The primary interest here is to identify the change in natural frequencies of these vibration modes.

**RESULTS AND DISCUSSION**

The modal analysis results are studied to identify the modes corresponding to \( n=1 \). As shown in figure 2, in the \( n = 1 \) mode the cross-section executes a translational motion transverse to the axis. This can happen in two ways, namely (a) translation of the cross section along the \( y \) direction (b) translation of the cross section along the \( z \) direction. Thus, there are two \( n=1 \) modes corresponding to these translations. In this article, we refer to these modes as \( n=1 \) (\( y \)) and \( n=1 \) (\( z \)), respectively.

Similarly, modal analysis results are studied to identify the modes corresponding to \( n=2 \). As shown in figure 2, in the \( n =2 \) mode the cross-section oscillated in the form of ellipse. Again this can happen in two ways, namely (a) the major axis of the ellipse along the \( y \) axis (b) the major axis of the ellipse along the \( z \) direction. In this article, we refer to these modes as \( n=2 \) (\( y \)) and \( n=2 \) (\( z \)), respectively. All the four modes described above are illustrated in figure 3.
Due to symmetry of the straight pipe, the natural frequencies of the pairs described in each of the earlier two paragraphs are identical. However, for bent pipes this symmetry is disturbed. Hence, we wish to study the variation of these natural frequencies.

![Mode shapes of a straight pipe.](image)

Figure 3: Mode shapes of a straight pipe.

The natural frequencies corresponding to the modes described above are tabulated. These results are non-dimensionalized with respect to the corresponding natural frequency of the straight pipe. The variation of the non-dimensional frequency with increasing bend angle is shown in Figure 4. Variation of natural frequencies with respect to increasing bend angle is different for \( n=1(y) \) and \( n=1(z) \) mode as shown in figure 4. As bend angle increases, it is seen from the figure that the natural frequency of the \( n=1(y) \) mode increases while the natural frequency of the \( n=1(z) \) mode decreases. The former has a more rapid rate of change than the later. Thus, the influence of bend angle is in the form of additional stiffness for the \( n=1(y) \) mode whereas for the \( n=1(z) \) mode the effect of bend angle is that of additional mass.

In case of \( n=2(y) \) and \( n=2(z) \) mode, cross section of pipe oscillates in the form of ellipse. It is observed that as the bend angle increases, rate of increase of natural frequencies is minimal. Variation of normalised natural frequency for the pair of \( n=2 \) mode is same because the oscillations in the form of ellipse along \( y \) and \( z \) directions are identical with the increase of bend angle.
CONCLUSIONS
Natural frequency analysis of bent pipes has been presented in this article. Finite Element Method using the commercial package ANSYS has been used for the calculations. The objective of the study is to understand the effect of bend angles on modal stiffness and modal masses. The investigation has been limited to two different modes of vibration for the straight pipe. The incorporation of bend entails a loss of axial symmetry and each of these two modes bifurcate into two modes for the bent pipe (giving a total of four modes). These are illustrated in figure 3. Further, variations in natural frequency for each of these modes are studied as a function of the bend angle. It is found that the beam mode in the plane of the bend angle has a comparatively sharp increase in natural frequency. In contrast, the beam mode in the other orthogonal plane shows a gradual decrease in natural frequency. Both these 2 modes, shows an equally increasing trend in the natural frequency. For the cases of increasing (or decreasing) natural frequencies, the incorporation of bend angle is thus analogous to additional stiffness (or mass) of the straight pipe. These results serve as a reference point for further analytical investigation of dynamic analysis of bent pipes as a geometric perturbation of straight pipes [6].

APPENDIX: VALIDATION OF THE FE ANALYSIS
Apart from the mesh convergence validation, the following exercise was done to ensure the accuracy of the FEM model. In beam theory, for fixed-fixed end condition with central point load, the deflection and natural frequencies can be calculated with the help of standard analytical formulae [4, 5]. Straight circular cylindrical shell with the same boundary condition and loading is analysed using ANSYS. The results obtained by the two methods are verified to be close. This validates the analysis for straight pipes and allows us to use the FEA model even for bent configurations. The results of these analyses are presented below.

Figure 4. Normalised natural frequency Vs Bend angle
When the central load of 20000N is applied, the deformation calculated with analytical formulae is 
\[ \delta = \frac{FL^3}{192EI} \approx 9.19 \times 10^{-4} \text{m}. \] For the same data, Ansys results were found to be 9.76 \times 10^{-4} \text{m}. The percentage error is therefore 6.17%. Similarly the natural frequency using well-known relations is 
\[ \omega = 22.37 \sqrt{\frac{EI}{\rho l^4}} \approx 887 \text{ rad/sec}. \] Ansys modal analysis results in this case is 135.11 Hz (\approx 849.26 \text{ rad/sec}). The percentage change in error of these values is 4.21%.

**REFERENCES:**


